

STATISTICS

Study Materials: Key Concepts, Practice Questions, Answers

Mathema
Contest

Statistics -Study guide and Practice Questions



Name: _____

Key Concepts

1. Descriptive Statistics

- **Measures of Central Tendency:** Mean, median, mode.
 - **Mean:** The average of a set of numbers.
 - **Median:** The middle value when a data set is ordered.
 - **Mode:** The value that appears most frequently in a data set.
- **Measures of Dispersion:** Range, variance, standard deviation.
 - **Range:** The difference between the highest and lowest values.
 - **Variance:** The average of the squared differences from the mean.
 - **Standard Deviation:** The square root of the variance, indicating how spread out the numbers are.

2. Probability

- **Basic Probability:** The likelihood of an event occurring, calculated as the number of favorable outcomes divided by the total number of possible outcomes.
 - **$P(A)$:** Probability of event A.
 - **$P(A \cup B)$:** Probability of A or B occurring (addition rule).
 - **$P(A \cap B)$:** Probability of both A and B occurring (multiplication rule).

3. Probability Distributions

- **Discrete Distributions:** Involve countable outcomes (e.g., binomial distribution).
 - **Binomial Distribution:** Describes the number of successes in a fixed number of independent trials.
 - Parameters: n (number of trials), p (probability of success).
 - Mean: $\mu = np$
 - Variance: $\sigma^2 = np(1 - p)$
- **Continuous Distributions:** Involve an infinite number of possible outcomes (e.g., normal distribution).

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- **Normal Distribution:** A bell-shaped curve, characterized by its mean (μ) and standard deviation (σ).
 - Standard Normal Distribution: A normal distribution with $\mu = 0$ and $\sigma = 1$

4. Inferential Statistics

- **Sampling Distributions:** The probability distribution of a statistic based on a random sample.
- **Confidence Intervals:** A range of values used to estimate the true population parameter.
 - Formula: $\bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}} \right)$, where $\bar{x}$ is the sample mean, z^* is the critical value, σ is the population standard deviation, and n is the sample size.
- **Hypothesis Testing:** A method for making inferences about population parameters.
 - Null Hypothesis (H_0): A statement of no effect or no difference.
 - Alternative Hypothesis (H_a): A statement that contradicts the null hypothesis.
 - Test Statistics: Measures how far the sample statistic is from the null hypothesis.
 - p-value: The probability of observing the test statistic or something more extreme under the null hypothesis.
 - Significance Level (α): The threshold for rejecting the null hypothesis.

5. Regression Analysis

- **Simple Linear Regression:** Models the relationship between two variables by fitting a linear equation.
 - Equation: $y = \beta_0 + \beta_1 x$
 - β_0 : Intercept, β_1 : Slope
 - R^2 : Coefficient of determination indicates, the proportion of variance in the dependent variable explained by the independent variable.

Practice Questions

1. Descriptive Statistics

- Calculate the mean, median, and mode for the following data set: [4, 8, 6, 5, 3, 8, 9].

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2. Probability

- If a card is drawn at random from a standard deck of 52 cards, what is the probability of drawing an Ace or a King?

3. Probability Distributions

- A coin is flipped 10 times. What is the probability of getting exactly 6 heads?

4. Inferential Statistics

- A sample of 25 students has a mean test score of 80 with a standard deviation of 5. Construct a 95% confidence interval for the true mean test score.

5. Regression Analysis

- Given the regression equation $y = 2 + 3x$, predict the value of y when $x = 4$.

Answers

1. Descriptive Statistics

- Data set: [4, 8, 6, 5, 3, 8, 9]
 - **Mean:** 6.14
 - **Median:** 6
 - **Mode:** 8

2. Probability

- Probability of drawing an Ace (4 out of 52) or a King (4 out of 52):
 - $P(\text{Ace} \cup \text{King}) = P(\text{Ace}) + P(\text{King}) = \frac{2}{13}$

3. Probability Distributions

- Binomial distribution: $n = 10$, $p = 0.5$, $k = 6$.
 - $P(X = 6) = 0.205$

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4. Inferential Statistics

- 95% Confidence interval: [78.04, 81.96]

5. Regression Analysis

- Predicted value of y is 14