

FUNCTIONS

Study Materials: Key Concepts, Practice Questions, Answers

Mathema
Contest

Functions -Study guide and Practice Questions



Name: _____

Key Concepts

1. Definition of a Function

- A function f from a set A to a set B assigns each element in A exactly one element in B .
 - **Notation:** $f: A \rightarrow B$.
 - $f(x)$ denotes the output of the function f for the input x .

2. Domain and Range

- **Domain:** The set of all possible input values x for the function f .
- **Range:** The set of all possible output values $f(x)$.

3. Types of Functions

- **Linear Function:** $f(x) = mx + b$, where m is the slope and b is the y-intercept.
- **Quadratic Function:** $f(x) = ax^2 + bx + c$, where $a \neq 0$.
- **Polynomial Function:** $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$
- **Rational Function:** $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$
- **Exponential Function:** $f(x) = a \cdot b^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.
- **Logarithmic Function:** $f(x) = \log_b(x)$, the inverse of the exponential function b^x .

4. Composition of Functions

- The composition of functions f and g is denoted by $(f \circ g)(x) = f(g(x))$.

5. Inverse Functions

- A function f has an inverse function f^{-1} if $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ for all x in the domain of f .
- The graph of f^{-1} is the reflection of the graph of f across the line $y = x$

6. Transformations of Functions

- **Vertical Shift:** $f(x) + k$ shifts the graph k units up if $k > 0$ or down if $k < 0$.

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- **Horizontal Shift:** $f(x - h)$ shifts the graph h units to the right if $h > 0$ or to the left if $h < 0$.
- **Vertical Stretch/Compression:** $af(x)$ stretches the graph vertically by a factor of a if $a > 1$ and compresses it if $0 < a < 1$.
- **Reflection:** $-f(x)$ reflects the graph across the x -axis, $f(-x)$ reflects the graph across the y -axis.

Practice Questions

1. Definition of a Function

- Determine whether the relation $\{(1, 2), (2, 4), (3, 6), (2, 5)\}$ is a function.

2. Domain and Range

- Find the domain and range of the function $f(x) = \sqrt{x - 1}$.

3. Composition of Functions

- Given $f(x) = 2x + 3$ and $g(x) = x^2$, find $(f \circ g)(2)$.

4. Inverse Functions

- Find the inverse of the function $f(x) = 3x - 4$

. Transformations of Functions

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- Describe the transformation of the function $f(x) = x^2$ to $g(x) = (x - 2)^2 + 3$.

Answers

1. Definition of a Function

- The relation is **not** a function because the element 2 in the domain is associated with two different elements (4 and 5) in the range.

2. Domain and Range

- Domain: $[1, \infty)$
- Range: $[0, \infty)$

3. Composition of Functions

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4. Inverse Functions

- $f^{-1}(x) = \frac{x+4}{3}$

5. Transformations of Functions

- A horizontal shift 2 units to the right and a vertical shift 3 units up of the parent function