

APPROXIMATION AND ERRORS

Study Materials: Key Concepts, Practice Questions, Answers

Mathema
Contest

Approx & Errors -Study guide and Practice Questions



Name: _____

Study Guide: Approximation and Errors

1. Introduction to Approximation and Errors

- **Approximation:** Process of finding a value close to the true value but easier to work with.
- **Errors:** Difference between the true value and the approximated value.

2. Types of Errors

- **Absolute Error (E_{abs}):** $|\text{True Value} - \text{Approximate Value}|$
- **Relative Error (E_{rel}):**
$$\frac{|\text{True Value} - \text{Approximate Value}|}{|\text{True Value}|}$$
- **Percent Error:** $\text{Relative Error} \times 100\%$

3. Significant Figures

- Reflect the precision of a measurement.
- Rules for determining significant figures:
 - All non-zero digits are significant.
 - Zeros between significant digits are significant.
 - Leading zeros are not significant.
 - Trailing zeros are significant if there is a decimal point.

4. Rounding Off Numbers

- If the digit to be dropped is greater than 5, increase the last retained digit by one.
- If the digit to be dropped is less than 5, do not change the last retained digit.
- If the digit to be dropped is exactly 5, round to the nearest even number.

5. Error Propagation

- When quantities with errors are combined, their errors combine.

- **Addition/Subtraction:**

$$E = E_1 + E_2 + \dots + E_n \quad \sqrt{E} = \sqrt{E_1^2 + E_2^2 + \dots + E_n^2}$$

- **Multiplication/Division:**

$$E = (E_1 A_1)^2 + (E_2 A_2)^2 + \dots + (E_n A_n)^2 \quad \sqrt{E} = \sqrt{E_1^2 A_1^2 + E_2^2 A_2^2 + \dots + E_n^2 A_n^2}$$

6. Truncation Errors

- Occur when an infinite process is approximated by a finite one (e.g., truncating a series).

7. Rounding Errors

- Occur due to rounding off numbers at various stages of calculations.

Practice Questions and Answers

Question 1

Calculate the absolute and relative error if the true value is 50.0 and the approximate value is 47.5.

Answer:

- Absolute Error: $|50.0 - 47.5| = 2.5$
- Relative Error:

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$$2.550.02.550.0$$

$$= 0.05 \text{ (or 5\%)}$$

Question 2

A length is measured as 25.0 cm with an uncertainty of ± 0.2 cm. What is the percent error?

Answer:

- Percent Error:

$$0.225.00.225.0$$

$$\times 100\% = 0.8\%$$

Question 3

Round the number 123.4567 to three significant figures.

Answer:

- Rounded Value: 123

Question 4

A calculation yields a value of 4.365. If this value needs to be rounded to two decimal places, what is the result?

Answer:

- Rounded Value: 4.37

Question 5

If the values 3.12 and 4.56 have absolute errors of 0.01 and 0.02, respectively, what is the absolute error of their sum?

Answer:

- Absolute Error of Sum:

$$0.01_2 + 0.02_2 \sqrt{=} 0.0001 + 0.000\sqrt{4} = 0.0005\sqrt{=} 0.02240.012 + 0.022 = 0.0001 + 0.0004 = 0.0005 = 0.0224$$

Question 6

A measurement is reported as

$$8.76 \times 10_3 \quad 8.76 \times 103$$

with an absolute error of

$$1.2 \times 10_2 \quad 1.2 \times 102$$

. What is the relative error?

Answer:

- Relative Error:

$$1.2 \times 10_2 \quad 8.76 \times 10_3 = 1208760 \approx 0.01371.2 \times 102 \quad 8.76 \times 103 = 1208760 \approx 0.0137$$

Question 7

Estimate the truncation error if the function

$$f(x) = e^x \quad f x = e x$$

is approximated by

$$1 + x + x^2_2! \quad 1 + x + x^22!$$

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for

$$x=0.1 \text{ , } x=0.1 \text{ .}$$

Answer:

- Exact Value:

$$e^{0.1} \approx 1.105170918 \quad e^{0.1} \approx 1.105170918$$

- Approximate Value:

$$1 + 0.1 + \frac{0.1^2}{2} = 1.1051 + 0.1 + \frac{0.1^2}{2} = 1.105$$

- Truncation Error:

$$|1.105170918 - 1.105| = 0.000170918 \quad 1.105170918 - 1.105 = 0.000170918$$

Question 8

If two measured values

$$A = 5.0 \pm 0.1$$

and

$$B = 0.3 \pm 0.2$$

are multiplied, what is the resulting value and its absolute error?

Answer:

- Resulting Value:

$$A \times B = 5.0 \times 0.3 = 1.5$$

- Relative Errors:

$$\frac{0.1}{5.0} = 0.02$$

and

$$\frac{0.2}{0.3} \approx 0.667$$

- Combined Relative Error:

$$0.02 + 0.667 \approx 0.687$$

- Absolute Error:

$$0.687 \times 1.5 = 1.03$$

- Result:

$$1.5 \pm 1.03$$

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Question 9

The true value of a quantity is 98.76, and its measured value is 97.8. Calculate the percent error.

Answer:

- Absolute Error:

$$|98.76 - 97.8| = 0.96$$

- Relative Error:

$$\frac{0.96}{98.76} \approx 0.0097$$

- Percent Error:

$$0.0097 \times 100\% \approx 0.97\%$$

Question 10

A series truncation yields an approximate value of 2.71828 for e. Calculate the truncation error.

Answer:

- Exact Value of e: 2.718281828...
- Truncation Error: $|2.718281828 - 2.71828| = 0.000001828$