

ALGEBRA II

Study Materials: Key Concepts, Practice Questions, Answers

Mathema
Contest

Algebra II -Study guide and Practice Questions



Name: _____

Key Concepts

1. Polynomial Functions

- Definition: A polynomial function is a function that can be expressed in the form $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where $(a_n, a_{n-1}, \dots, a_1, a_0)$ are constants and n is a non-negative integer.
- Example: $P(x) = 2x^3 - 4x^2 + x - 5$

2. Quadratic Functions

- Definition: A quadratic function is a polynomial of degree 2, typically in the form $f(x) = ax^2 + bx + c$
- Vertex Form: $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex.
- Example: $f(x) = 3x^2 + 6x + 2$

3. Exponential Functions

- Definition: An exponential function is of the form $f(x) = a \cdot b^x$, where a and b are constants, and b is positive and not equal to 1.
- Example: $f(x) = 2 \cdot 3^x$

4. Logarithmic Functions

- Definition: A logarithmic function is the inverse of an exponential function, typically in the form $f(x) = \log_b(x)$, where b is the base.
- Example: $f(x) = \log_2(x)$

5. Solving Quadratic Equations

- Methods:
 - Factoring: Express the quadratic as a product of binomials.
 - Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Completing the Square: Rewrite the quadratic in the form $(x - p)^2 = q$

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Worked Examples

1. Polynomial Functions

Example **Simplify**: $(2x^3 - 4x^2 + x - 5) + (3x^3 + 2x^2 - 4x + 7)$

Solution:

- Combine the like terms.

$$\begin{aligned} & (2x^3 - 4x^2 + x - 5) + (3x^3 + 2x^2 - 4x + 7) \\ &= (2x^3 + 3x^3) + (-4x^2 + 2x^2) + (x - 4x) + (-5 + 7) \\ &= 5x^3 - 2x^2 - 3x + 2 \end{aligned}$$

Therefore, the simplified form is: $(2x^3 - 4x^2 + x - 5) + (3x^3 + 2x^2 - 4x + 7) = 5x^3 - 2x^2 - 3x + 2$

2. Quadratic Functions

Example: Convert $f(x) = 2x^2 - 8x + 6$ to vertex form.

Solution:

- Complete the square. $f(x) = 2x^2 - 8x + 6 = 2(x^2 - 4x) + 6$
$$\begin{aligned} &= 2(x^2 - 4x + 4 - 4) + 6 \\ &= 2((x - 2)^2 - 4) + 6 \\ &= 2(x - 2)^2 - 8 + 6 \\ &= 2(x - 2)^2 - 2 \end{aligned}$$

Therefore, the vertex form is: $f(x) = 2(x - 2)^2 - 2$

3. Exponential Functions

Example: Solve for x in $3^x = 81$

Solution:

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- Express 81 as a power of 3: $81 = 3^4$
- Set the exponents equal to each other : $3^x = 3^4$, *implies* $x = 4$

Therefore, the solution is: $x = 4$

4. Logarithmic Functions

Example: Solve for x in $\log_2(x) = 5$

Solution:

- Rewrite the equation in exponential form.

$$\log_2(x) = 5, \text{ implies } 2^5 = x$$

- Calculate the value : $x = 32$

Therefore, the solution is: $x = 32$

5. Solving Quadratic Equations

Example: Solve $3x^2 - 2x - 5 = 0$ using the quadratic formula.

Solution:

- Identify a , b , and c : $a = 3$, $b = -2$, $c = -5$
- Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Substitute the values:

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3} \\ &= \frac{2 \pm \sqrt{4 + 60}}{6} \\ &= \frac{2 \pm \sqrt{64}}{6} \\ &= \frac{2 \pm 8}{6} \end{aligned}$$

- Solve for the two possible values of x .

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$$x = \frac{2+8}{6} = \frac{10}{6} = \frac{5}{3}$$

$$x = \frac{2-8}{6} = \frac{-6}{6} = -1$$

Therefore, the solutions are: $x = \frac{5}{3}$ or $x = -1$

6. Graphing Quadratics

Example: Graph the function $f(x) = -x^2 + 4x - 3$

Solution:

- Find the vertex using $(h = -\frac{b}{2a})$, Substitute the values ($b = 4$) and ($a = -1$): $h = -\frac{4}{2(-1)} = 2$
- Substitute h back into the function to find k : $k = -2^2 + 4(2) - 3 = -4 + 8 - 3 = 1$
- The vertex is $(2,1)$, and the parabola opens downwards because the coefficient of (x^2) is negative.

Therefore, the vertex form is $f(x) = -(x - 2)^2 + 1$

7. Systems of Equations

Example: Solve the system:

$$\begin{cases} 2x + 3y = 6 \\ 4x - y = 5 \end{cases}$$

Solution:

- Solve the second equation for y : $y = 4x - 5$
- Substitute y into the first equation.

$$\begin{aligned} 2x + 3(4x - 5) &= 6 \\ 2x + 12x - 15 &= 6 \\ 14x - 15 &= 6 \\ 14x &= 21, x = 1.5 \end{aligned}$$

- Substitute x back into $y = 4x - 5$

$$y = 4(1.5) - 5 = 6 - 5 = 1$$

Therefore, the solution is: $x = 1.5$, $y = 1$

8. Simplifying Rational Expressions

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Example: Simplify $\frac{x^2-9}{x^2-6x+9}$

Solution:

- Factor the numerator and the denominator.

$$\frac{x^2 - 9}{x^2 - 6x + 9} = \frac{(x - 3)(x + 3)}{(x - 3)^2}$$

- Cancel out the common factor $(x - 3)$

$$\frac{(x - 3)(x + 3)}{(x - 3)(x - 3)} = \frac{x + 3}{x - 3}$$

Therefore, the simplified form is: $\frac{x+3}{x-3}$

9. Solving Exponential Equations

Example: Solve for x in $2e^{3x} = 10$

Solution:

- Divide both sides by 2 : $e^{3x} = 5$
- Take the natural logarithm of both sides: $3x = \ln(5)$
- Solve for x : $x = \frac{\ln(5)}{3}$

Therefore, the solution is: $x = \frac{\ln(5)}{3}$

10. Logarithmic Properties

Example: Simplify $\log_3(27) + \log_3(9)$

Solution:

- Use the properties of logarithms : $\log_3(27) = 3$ and $\log_3(9) = 2$
- Add the logarithms : $\log_3(27) + \log_3(9) = 3 + 2 = 5$

Alternate Method :

- Use the properties of logarithms to combine the logs : $\log_3(27) + \log_3(9)$
 - Apply the product rule: $\log_b(m) + \log_b(n) = \log_b(m \cdot n)$
- Therefore, $\log_3(27) + \log_3(9) = \log_3(27 \cdot 9)$

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- *Simplify inside the logarithm:* $\log_3(27 \cdot 9) = \log_3(243)$
- *Express 243 as a power of 3:* $243 = 3^5$
- *Use the power rule:* $\log_b(b^k) = k$

Therefore, $\log_3(3^5) = 5$

Practice Questions

1. Polynomial Functions:

- Simplify: $(2x^3 - 4x^2 + x - 5) + (3x^3 + 2x^2 - 4x + 7)$

2. Quadratic Functions:

- Convert to vertex form: $f(x) = 2x^2 - 8x + 6$

3. Exponential Functions:

- Solve for x : $3^x = 81$

4. Logarithmic Functions:

- Solve for x : $\log_2(x) = 5$

5. Solving Quadratic Equations:

- Solve using the quadratic formula: $3x^2 - 2x - 5 = 0$

6. Graphing Quadratics:

- Graph the function: $f(x) = -x^2 + 4x - 3$

7. Systems of Equations:

- Solve the system:

$$\begin{cases} 2x + 3y = 6 \\ 4x - y = 5 \end{cases}$$

8. Simplifying Rational Expressions:

- Simplify: $\frac{x^2-9}{x^2-6x+9}$

9. Solving Exponential Equations:

- Solve for x : $2e^{3x} = 10$

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10. Logarithmic Properties:

- Simplify: $\log_3(27) + \log_3(9)$

Answers

1. Polynomial Functions: $(2x^3 - 4x^2 + x - 5) + (3x^3 + 2x^2 - 4x + 7) = 5x^3 - 2x^2 - 3x + 2$
2. Quadratic Functions: $f(x) = 2(x - 2)^2 - 2$
3. Exponential Functions : $x = 4$ (since $3^4 = 81$)
4. Logarithmic Functions : $x = 32$ (since $2^5 = 32$)
5. Solving Quadratic Equations : $x = \frac{2 \pm \sqrt{64}}{6} \rightarrow x = \frac{2 \pm 8}{6} \rightarrow x = \frac{10}{6}$ or $x = \frac{-6}{6} \rightarrow x = \frac{5}{3}$ or $x = -1$
6. Graphing Quadratics : The vertex is at (2,1) and the parabola opens downwards
7. Systems of Equations : $x = 1, y = 43x = 1, y = \frac{4}{3}x = 1, y = 34$
8. Simplifying Rational Expressions : $\frac{(x-3)(x+3)}{(x-3)^2} = \frac{x+3}{x-3}$
9. Solving Exponential Equations : $2e^{3x} = 10 \rightarrow e^{3x} = 5 \rightarrow 3x = \ln(5) \rightarrow x = \frac{\ln(5)}{3}$
10. Logarithmic Properties : $\log_3(27) + \log_3(9) = 3 + 2 = 5$